Math 150, *Lecture Notes- Bonds* Name____________

Section 2.5 *Implicit Differentiation*

Implicit and Explicit Functions

Up to this point in the text, most functions have been expressed in **explicit form.** For example, in the equation

$y = 3x^2 - 5$ Explicit form

the variable y is explicitly written as a function of x. Some functions, however, are only implied by an equation. For instance, the function $y = 1/x$ is defined **implicitly** by the equation $xy = 1$. Suppose you were asked to find dy/dx for this equation. You could begin by writing y explicitly as a function of x and then differentiating.

Implicit Form	*Explicit Form*	*Derivative*
$xy = 1$	$y = \dfrac{1}{x} = x^{-1}$	$\dfrac{dy}{dx} = -x^{-2} = -\dfrac{1}{x^2}$

This strategy works whenever you can solve for the function explicitly. You cannot, however, use this procedure when you are unable to solve for y as a function of x. For instance, how would you find dy/dx for the equation

$x^2 - 2y^3 + 4y = 2$

where it is very difficult to express y as a function of x explicitly? To do this, you can use **implicit differentiation.**

To understand how to find dy/dx implicitly, you must realize that the differentiation is taking place *with respect to x*. This means that when you differentiate terms involving x alone, you can differentiate as usual. However, when you differentiate terms involving y, you must apply the Chain Rule, because you are assuming that y is defined implicitly as a differentiable function of x.

Ex.1 Differentiating with Respect to x.

a. $\dfrac{d}{dx}[x^3] = 3x^2$ Variables agree: use Simple Power Rule.

Variables agree

b. $\dfrac{d}{dx}[\underbrace{y^3}_{u^n}] = \underbrace{3y^2}_{nu^{n-1}}\underbrace{\dfrac{dy}{dx}}_{u'}$ Variables disagree: use Chain Rule.

Variables disagree

c. $\dfrac{d}{dx}[x + 3y] = 1 + 3\dfrac{dy}{dx}$ Chain Rule: $\dfrac{d}{dx}[3y] = 3y'$

d. $\dfrac{d}{dx}[xy^2] = x\dfrac{d}{dx}[y^2] + y^2\dfrac{d}{dx}[x]$ Product Rule

$= x\left(2y\dfrac{dy}{dx}\right) + y^2(1)$ Chain Rule

$= 2xy\dfrac{dy}{dx} + y^2$ Simplify.

Guidelines for Implicit Differentiation

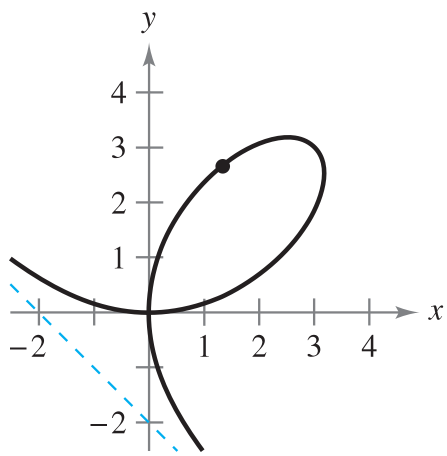
1. Differentiate both sides of the equation *with respect to x*.
2. Collect all terms involving dy/dx on the left side of the equation and move all other terms to the right side of the equation.
3. Factor dy/dx out of the left side of the equation.
4. Solve for dy/dx.

Ex.2 Find $\frac{dy}{dx}$, given that $2x^3+3y^3=64$.

Ex.3 Find $\frac{dy}{dx}$, given that $x^2y+y^2x=-2$.

Ex.4 Find $\frac{dy}{dx}$ and evaluate the derivative at $(2,2)$, given that $y^3-x^2=4$.

Ex.5 Find the equation of the tangent line to the graph of $x^3 + y^3 - 6xy = 0$ at $\left(\frac{4}{3}, \frac{8}{3}\right)$.



Ex.6 Find $\dfrac{d^2y}{dx^2}$, given that $x^2 - y^2 = 36$.